An Exact Algorithm for the Multi-Vehicle Routing Problem with Stochastic Demands

Walter Rei
CIRRELT
and
Département de management et technologie
Université du Québec à Montréal

and

Michel Gendreau
CIRRELT
and
Département de mathématiques et génie industriel
École Polytechnique de Montréal

Eindhoven University of Technology
School of Industrial Engineering
November 4 2009
Presentation Outline

- \( m \)-VRP with Stochastic Demands
  - General Description
  - Model
  - Related work

- General Solution Approach
  - 0-1 Integer L-Shaped Algorithm
  - Applying the method to the Stochastic \( m \)-VRP
  - Limitations of the Method

- Improving the L-Shaped Method
  - Local Branching Cuts
    - General Approach
    - Valid Inequalities
    - Application to the Stochastic \( m \)-VRP
    - Results
Presentation Outline (cont’d)

- Improving the L-Shaped Method (cont’d)
  - LBF Strategies
    - Partial Routes
    - Classical Cuts
    - Improved Cuts
    - Separation procedures
    - Results

- Research Perspectives
$m$-VRP with Stochastic Demands

- **General Description**
  - Stochastic $m$-VRP: a fleet of $m$ capacitated vehicles must deliver (collect) unknown demands to (from) a set of customers:

    \[ G(V, E): \text{an undirected graph,} \]
    \[ \text{where } V = \{v_1, \ldots, v_n\} \text{ and } E = \{(v_i, v_j) : v_i, v_j \in V, i < j\} \]
    \[ C = (c_{ij}) \text{ travel cost matrix} \]
    \[ v_1 = \text{depot} \]
    \[ D = \text{capacity of each vehicle} \]

  - Demands are revealed when vehicle arrives at a client given location:

    \[ \xi_i = \text{demand of client } i, \text{ where } i = 2, \ldots, n \]

  - Recourse ⇒ return to depot if vehicle cannot fulfill customer demand
-VRP with Stochastic Demands (cont’d)

General Description (cont’d)

Apply the \textit{a priori} approach:

Bertsimas, Jaillet and Odoni (1990)

<table>
<thead>
<tr>
<th>1\textsuperscript{st} Stage</th>
<th>Build a plan</th>
<th>find $m$ routes that visit all clients once</th>
</tr>
</thead>
<tbody>
<tr>
<td>2\textsuperscript{nd} Stage</td>
<td>Use the plan</td>
<td>Run the routes for a given time</td>
</tr>
</tbody>
</table>

Objective:
- Routing cost + Average recourse cost

Constraints:
- Restrict the total expected demand routes:
The expected demand of each route $\leq D$
- Obtain more balanced routes
\( m \)-VRP with Stochastic Demands (cont’d)

**Model**

**Variables:**
\[
\begin{align*}
    x_{ij} &= \{0, 1\} & \text{for } i, j > 1 \\
    x_{1j} &= \{0, 1, 2\} & \text{for } j > 1
\end{align*}
\]

**Objective:**
\[
\min \sum_{i<j} c_{ij} x_{ij} + Q(x)
\]

**Constraints:**
\[
\sum_{j=2}^{n} x_{1j} = 2m \\
\sum_{i<k} x_{ik} + \sum_{j>k} x_{kj} = 2, \quad k = 2, \ldots, n \\
\sum_{v_i, v_j \in S} x_{ij} \leq |S| - \left[ \sum_{v_i \in S} \mathbf{E}(\xi_i)/D \right], \quad S \subset V \setminus \{v_1\}, 2 \leq |S| \leq n - 2 \\
    x_{ij} = \{0, 1\}, \quad i, j > 1 \\
    x_{1j} = \{0, 1, 2\} \quad j > 1
\]
$m$-VRP with Stochastic Demands (cont’d)

- Related Work
  - Gendreau, Laporte and Séguin (1995):
    - 1st implementation of the L-Shaped algorithm on the problem
  - Hjørring and Holt (1999):
    - Problem considered: 1-VRP with stochastic demands
    - New cuts $\Rightarrow$ partial routes
    - Improvement of the general lower bound
  - Laporte, Louveaux and Van hamme (2002):
    - Improvement of the general lower bound for the $m$-VRP with stochastic demands
    - Generalization of the partial route cuts
    - Greedy separation procedure for cuts
  - Christiansen and Lysgaard (2007):
    - 1st branch-and-price algorithm for the $m$-VRP with stochastic demands
General Solution Approach

- 0-1 Integer L-Shaped Algorithm
  - Method proposed by Laporte and Louveaux (1993)
  - Variant of branch-and-cut
    - Assumption 1: \( Q(x) \) is computable
    - Assumption 2: There exists a finite value \( L = \) general lower bound for the recourse function

Let \( x \in X = \overline{X} \cap \{0, 1\}^{n_1} \) define the 1st stage decisions

Define current master problem:

\[
\begin{align*}
\text{Min} & \quad c^T x + \Theta \\
\text{s.t.} & \quad Ax = b \\
& \quad D_k x \geq d_k, \ k = 1, \ldots, s, \\
& \quad E_l x + \Theta \geq e_l, \ l = 1, \ldots, t, \\
& \quad 0 \leq x \leq 1, \ \Theta \in \mathbb{R}.
\end{align*}
\]
General Solution Approach (cont’d)

- 0-1 Integer L-Shaped Algorithm (cont’d)
  - (3): Feasibility cuts ⇒ induce feasible values of \( x \)
    
    Valid set: \( \exists s \) such that \( x \in \overline{X} \) if and only if \( \{D_kx \geq d_k, \ k = 1, \ldots, s\} \)
    
    Problem dependent

- (4): Optimality cuts ⇒ express possible feasible values of \( Q(x) \)

  Valid set: if: \( \forall x \in X, (x, \Theta) \in \{(x, \Theta) \mid E_lx + \Theta \geq e_l, \ l = 1, \ldots, t\} \)
  
  implies \( \Theta \geq Q(x) \)

Let the \( r - th \) feasible solution be:

- \( x_i = 1, i \in S_r \) and \( x_i = 0, i \notin S_r \)
- \( \Theta_r = \) recourse value

\[
\Theta \geq (\Theta_r - L) \left( \sum_{i \in S_r} x_i - \sum_{i \notin S_r} x_i \right) - (\Theta_r - L)(|S_r| - 1) + L
\]
General Solution Approach (cont’d)

- 0-1 Integer L-Shaped Algorithm (cont’d)

  Algorithm:

  **Step 0:** Set $\nu = 0$, $\Theta \geq L$ and $\overline{z} = \infty$

  **Step 1:** Select a pendant node from the list. If none exists STOP

  **Step 2:** Set $\nu = \nu + 1$ and solve current master problem ⇒ $(x^\nu, \Theta^\nu)$

  **Step 3:** Search for violated feasibility cuts:
  - If one is found ⇒ (3) + current master problem, return to **Step 2**
  - Otherwise ⇒ If $c^\top x^\nu + \Theta^\nu \geq \overline{z}$ fathom node, return to **Step 1**

  **Step 4:** If $x^\nu$ is not integer ⇒ Branch, return to **Step 1**

  **Step 5:** Compute $Q(x^\nu)$, set $z^\nu = c^\top x^\nu + Q(x^\nu)$
  - If $z^\nu < \overline{z}$ ⇒ $\overline{z} = z^\nu$

  **Step 6:**
  - If $\Theta^\nu \geq Q(x^\nu)$ ⇒ fathom node, return to **Step 1**
  - Otherwise ⇒ (4) + current master problem, return to **Step 2**
General Solution Approach (cont’d)

Applying the Method to Stochastic $m$-VRP

Assumptions:

Assumption 1: $Q(x)$ is computable

Dror, Laporte and Trudeau (1989)

Given a feasible solution $x$:

- $Q(x) = \sum_{k=1}^{m} \min\{Q^{k,1}, Q^{k,2}\}$

Given additional assumptions:

- Goods are divisible: $\xi_j \sim N(\mu_j, \sigma_j)$, $j = 2, \ldots, n$
- Demands are independent

Assumption 2: there exists a finite value $L$

Laporte, Louveaux and Van hamme (2002) $\Rightarrow$ measure $L$ based on the probability of failure on each route taken separately
General Solution Approach (cont’d)

- Applying the Method to Stochastic $m$-VRP (cont’d)
  
  - Feasibility cuts ⇒ induce feasible values of $x$

  A priori solution $x$ ⇒ a set of $m$ routes such that the expected demand of each route $\leq D$

  Apply cutting strategies from the deterministic version:

  Considering: $\xi_i = \mu_i$, for $i = 2, \ldots, n$

  Lysgaard, Letchford and Eglese (2004):
  
  - Capacity inequalities
  - Framed capacity inequalities
  - Strengthened comb inequalities
  - Multistar and partial multistar inequalities
  - Hypotour inequalities
General Solution Approach (cont’d)

- Limitations of the Method
  - Main problem: approximation of $Q(x)$
  - Information provided by optimality cuts:
    - very local
      - (4) only bound $Q(x)$ for the feasible solutions encountered
  - In *current master problem* the value of recourse:
    - $L$
    - Subset of optimality constraints
  - Risk of enumeration
Improving the L-Shaped Method

Local Branching Cuts

General Approach

Method proposed by Fischetti and Lodi (2003)
Based on the use of generic solvers (CPLEX)
Separation of the feasible region:

- let $x^0$ = feasible solution to the original problem,
- let $N_1 = \{1, \ldots, n_1\}$ and $S_0 = \{j \in N_1 \mid x^0_j = 1\}$,
- let $\kappa = \text{positive integer}$,
- The Hamming distance:

$$\Delta(x, x^0) = \sum_{j \in S_0} (1 - x_j) + \sum_{j \in N_1 \setminus S_0} x_j$$

- Left branch : $\Delta(x, x^0) \leq \kappa$
- Right branch : $\Delta(x, x^0) \geq \kappa + 1$
Improving the L-Shaped Method (cont’d)

Local Branching Cuts (cont’d)

General Approach (cont’d)

Local Branching subproblem:

\[ \Delta(x, x^0) \leq \kappa \]
Improving the L-Shaped Method (cont’d)

Local Branching Cuts (cont’d)

General Approach (cont’d)

Possible outcomes for the LB subproblem:

- subproblem is feasible (we obtain a solution $x^1$)
- subproblem is infeasible

If the subproblem is infeasible

- increase the size of the subproblem

If subproblem is feasible and $c^T x^1 + Q(x^1) \geq c^T x^0 + Q(x^0)$

- add $\Delta(x, x^1) \geq 1$
- increase the size of the subproblem

Otherwise (i.e., $c^T x^1 + Q(x^1) < c^T x^0 + Q(x^0)$)

- branching strategy
Improving the L-Shaped Method (cont’d)

Local Branching Cuts (cont’d)

General Approach (cont’d)

Subproblem size increase:

\[ \Delta(x, x^0) \leq \kappa + \left\lceil \frac{\kappa}{2} \right\rceil \]
Improving the L-Shaped Method (cont’d)

- Local Branching Cuts (cont’d)
  - General Approach (cont’d)

Branching step:

\[ \Delta(x, x^0) \leq \kappa \]
\[ \Delta(x, x^0) \geq \kappa + 1 \]

\[ \Delta(x, x^1) \leq \kappa \]

...
Improving the L-Shaped Method (cont’d)

Local Branching Cuts (cont’d)

Valid Inequalities

Define the two following subproblems:

\[
(P_n) \quad \text{Min} \quad c^\top x + Q(x) \quad \quad (P_n) \quad \text{Min} \quad c^\top x + Q(x)
\]
\[
s.t. \quad Ax = b \quad \quad s.t. \quad Ax = b
\]
\[
\Delta(x, x^i) \geq \kappa_i, \quad i \in I^n
\]
\[
\Delta(x, x^n) \leq \kappa_n
\]
\[
x \in X
\]

\[\cdot \quad I^n = \text{set of 0-1 vectors that may or may not represent feasible 1\textsuperscript{st} stage solutions}\]

\[\cdot \quad P_n \text{ for } n = 1, \ldots, m \Rightarrow \text{a local branching descent}\]

\[\cdot \quad \Theta_n = \text{a lower bound on } Q(x) \text{ for the subregion associated with } P_n, n = 1, \ldots, m\]
Improving the L-Shaped Method (cont’d)

Local Branching Cuts (cont’d)

Valid Inequalities (cont’d)

Redefine the current master problem in the following way:

\[
\begin{align*}
\text{Min} & \quad c^T x + \Theta \\
\text{s.t.} & \quad Ax = b \quad (6) \\
D_k x & \geq d_k, \quad k = 1, \ldots, s, \quad (7) \\
E_l x + \Theta & \geq e_l, \quad l = 1, \ldots, t, \quad (8) \\
\Delta(x, x^i) & \geq 1, \quad i \in \bigcup_{n=1}^{m} I^n, \quad (9) \\
0 & \leq x \leq 1, \quad \Theta \in \mathbb{R} \quad (10)
\end{align*}
\]

Constraint \( \Delta(x, x^i) \geq 1 \) eliminates locally vector \( x^i \)

- If \( x^i \) is feasible \( \Rightarrow (10) \) is a weak optimality cut
- If \( x^i \) is infeasible \( \Rightarrow (10) \) serves as a feasibility cut
Proposition
Let $P_n, n = 1, \ldots, m$, define a local branching descent and $\Theta_n, n = 1, \ldots, m$, be valid lower bounds on the recourse value for each of the subproblems in the descent, then the following system of equations defines a set of valid inequalities for problem (6)-(11):

\begin{align*}
\Theta & \geq L + (\Theta_n - L)w_n, \ n = 1, \ldots, m \\
\kappa_n - \Delta(x, x^n) & \leq n_1 \sum_{j=1}^{n} w_j, \ n = 1, \ldots, m \\
\Delta(x, x^n) - \kappa_n & \leq (1 - w_n)n_1, \ n = 1, \ldots, m \\
\sum_{j=1}^{m} w_j & \leq 1 \\
w_n & \in \{0, 1\}, \ n = 1, \ldots, m
\end{align*}
Improving the L-Shaped Method (cont’d)

Local Branching Cuts (cont’d)

Valid Inequalities (cont’d)

System (12)-(16) bounds the value of $Q(x)$ following the local branching descent.

If $\hat{x}$ is a solution to the current master problem, then a valid lower bound on $Q(\hat{x})$ is provided in the first $P_n, n = 1, \ldots, m$ for which $\hat{x}$ is feasible.

Advantages:

- If $\overline{\Omega}_n > L, n = 1, \ldots, m$, then (12)-(18) offers a better description.
- (12)-(16) bounds $Q(x)$ in different subregions of $X$.

Limitations:

- System of inequalities which makes the current master problem harder to solve.
Improving the L-Shaped Method (cont’d)

Local Branching Cuts (cont’d)

Application to the Stochastic $m$-VRP

Case: Stochastic 1-VRP:

Separation Strategy:

Route building:
- CVRPSEP package $\Rightarrow$ Lysgaard, Letchford and Eglese (2004)

Lower bounding functionnals:
- Partial route cuts $\Rightarrow$ Hjorring and Holt (1999)

Local branching descent:
- Tightening the lower bound
Improving the L-Shaped Method (cont’d)

Local Branching Cuts (cont’d)

Application to the Stochastic $m$-VRP (cont’d)

Implementing local branching:
- Global cuts ⇒ current master problem (root node)
- Local cuts ⇒ current master problem (subproblem in search tree)
- Lower bound ⇒ current master problem + integrality + separation procedures

Note:

Global cuts can be used throughout the solution process but local cuts can only be used in the node of the search tree of the associated subproblem.
Improving the L-Shaped Method (cont’d)

Local Branching Cuts (cont’d)

Results

Test problems:

- Problem generator ⇒ Hjörring and Holt (1999)
- Problems of size \( n = 20, 30, 40, \ldots, 90 \) were generated
- For each size, five instances were generated for which \( \bar{f} = 95\%, 97.5\%, \ldots, 110\% \) (total: 280 instances)

\[
\bar{f} = \frac{\sum_{i=1}^{n} \mu_i}{D} \Rightarrow \text{filling coefficient}
\]

Algorithms:

- Standard: L-shaped without local branching
- LB: L-shaped + local cuts (\( m = 3 \) and \( \kappa = 4, 6 \) and 8)
- LB1: L-shaped + global cuts (\( m = 3 \) and \( \kappa = 4, 6 \) and 8)

Runs: maximum time of 1200 seconds
Improving the L-Shaped Method (cont’d)

Local Branching Cuts (cont’d)

Results (cont’d)

Standard vs. LB-4

On easy instances (< 60): the two algorithms are equivalent
On medium instances ([60, 1200]): 30 instances

<table>
<thead>
<tr>
<th>Times</th>
<th>Standard</th>
<th>LB-4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average</td>
<td>359.27 sec.</td>
<td>35.35 sec.</td>
</tr>
<tr>
<td>Total</td>
<td>10778.15 sec.</td>
<td>1060.53 sec.</td>
</tr>
</tbody>
</table>

On hard instances: 74 instances

- 24 instances solved by LB-4
- Average gap: Standard 3.42% vs. LB-4 2.04%

L-shaped algorithm is either faster or obtains better results when local branching is applied
Improving the L-Shaped Method (cont’d)

Local Branching Cuts (cont’d)

Results (cont’d)

LB vs. LB1

Variations in $\kappa$

Number of instances solved using LB:
- LB-4: 230
- LB-6: 231
- LB-8: 231

Number of instances solved using LB1:
- LB1-4: 217
- LB1-6: 223
- LB1-8: 221
Improving the L-Shaped Method (cont’d)

Local Branching Cuts (cont’d)

Results (cont’d)

LB vs. LB1 (cont’d)

Variations in maximum time

<table>
<thead>
<tr>
<th>Max. Time</th>
<th>LB</th>
<th>LB1</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>sol.</td>
<td>not sol.</td>
</tr>
<tr>
<td>1200</td>
<td>12/228.24</td>
<td>32/2.42%</td>
</tr>
<tr>
<td>2400</td>
<td>15/595.33</td>
<td>31/2.49%</td>
</tr>
<tr>
<td>3600</td>
<td>17/884.89</td>
<td>29/2.44%</td>
</tr>
<tr>
<td>4800</td>
<td>18/1078.25</td>
<td>29/2.52%</td>
</tr>
<tr>
<td>6000</td>
<td>18/1078.25</td>
<td>29/2.45%</td>
</tr>
</tbody>
</table>

Table: Results on hard instances: LB vs. LB1
Improving the L-Shaped Method

- LBF Strategies

Initially proposed by Hjorring and Holt (1999):
Bound the value of $Q(x)$ using Partial route cuts

- Partial Route

Illustration:

A series of chains and unstructured sets that are:

- Connected
- Begin and end at the depot
LBF Strategies (cont’d)

Classical Cuts

Let:

$S = (v_1, \ldots, v_S) \Rightarrow \text{Chain}$

$T = (v_1, \ldots, v_T) \Rightarrow \text{Chain}$

$U \Rightarrow \text{unstructured set such that:}$

- $U \cap S = \{v_S\}$
- $U \cap T = \{v_T\}$

$R = S \cup T \cup U$
Improving the L-Shaped Method (cont’d)

LBF Strategies (cont’d)

Classical Cuts (cont’d)

Bound the partial route:

Create node $v_0$ such that:

- $\xi_0 = \sum_{v_i \in U \setminus \{v_S, v_T\}} \xi_i$
- $c_{10} = \min_{v_i \in U \setminus \{v_S, v_T\}} \{c_{1i}\}$

Using node $v_0$, consider the following route:

$(v_1, \ldots, v_S, v_0, v_T, \ldots, v_1)$

By calculating $Q(x)$ this route, one obtains a lower bound $P$ on all routes that share chains $S$ and $T$ and where set $U$ is undefined.
Improving the L-Shaped Method (cont’d)

LBF Strategies (cont’d)

Classical Cuts (cont’d)

To obtain the cut:

Set: \( W(x) = \sum_{(v_i,v_j) \in S} x_{ij} + \sum_{(v_i,v_j) \in T} x_{ij} + \sum_{v_i,v_j \in U} x_{ij} - |R| + 1 \)

For 1 vehicle:

\[ \Theta \geq L + (P - L)W(x) \]

For \( m \) vehicles:

\[ \Theta \geq L + (P - L)\left( \sum_{h=1}^r W_h(x) - r + 1 \right) \]
Improving the L-Shaped Method (cont’d)

LBF Strategies (cont’d)

Improved Cuts

Let:

\[ S^1 = (v_1, \ldots, v_{S1}) \implies \text{Chain no.1} \]
\[ S^2 = (v'_{S2}, \ldots, v''_{S2}) \implies \text{Chain no.2} \]
\[ S^3 = (v_1, \ldots, v_{S3}) \implies \text{Chain no.3} \]
\[ U^1 \implies \text{Unstructured set no.1} \]
\[ U^2 \implies \text{Unstructured set no.2} \]
Improving the L-Shaped Method (cont’d)

LBF Strategies (cont’d)

Improved Cuts (cont’d)

Bound the partial:

Create nodes $v_0^1$ and $v_0^2$ such that:

- $\xi_0^1 = \sum_{v_i \in U_1 \setminus \{v_{S_1}, v_{S_2}'\}} \xi_i$
- $c_{10^1} = \min_{v_i \in U_1 \setminus \{v_{S_1}, v_{S_2}'\}} \{c_{1i}\}$
- $\xi_0^2 = \sum_{v_i \in U_2 \setminus \{v''_{S_2}, v_{S_3}\}} \xi_i$
- $c_{10^2} = \min_{v_i \in U_2 \setminus \{v''_{S_2}, v_{S_3}\}} \{c_{1i}\}$

Using nodes $v_0^1$ and $v_0^2$, consider the following route:

$(v_1, \ldots, v_{S_1}, v_0^1, v_{S_2}', \ldots, v''_{S_2}, v_{S_3}, v_0^2, \ldots, v_1)$

By calculating $Q(x)$ on this route, one obtains a lower bound $\hat{P}$ on all routes that share all previous chains and where all previous sets are undefined.
Improving the L-Shaped Method (cont’d)

LBF Strategies (cont’d)

Improved Cuts (cont’d)

To obtain the cut:

Set: \( \hat{W}(x) = 3 \sum_{k=1}^{3} \sum_{(v_i, v_j) \in S^k} x_{ij} + 2 \sum_{k=1}^{2} \sum_{v_i, v_j \in U^k} x_{ij} - |R| + 1 \)

For 1 vehicle:

\[ \Theta \geq L + (\hat{P} - L) \hat{W}(x) \]

For \( m \) vehicles:

\[ \Theta \geq L + (\hat{P} - L) \left( \sum_{h=1}^{r} \hat{W}_h(x) - r + 1 \right) \]

Note: the lower bound provided by the cut is improved given that \( \hat{P} \geq P \)
Improving the L-Shaped Method (cont’d)

LBF Strategies (cont’d)

Separation procedures

A solution \((x^\nu, \Theta^\nu)\) can be eliminated using and LBF cut if:

- \(\exists h\) such that \(W_h(x^\nu) = 1\) or \(\hat{W}_h(x^\nu) = 1\)
- \(P \geq \Theta^\nu\) or \(\hat{P} \geq \Theta^\nu\)

Heuristic separation approach:

- Laporte, Louveaux and Van hamme (2002)
- Greedy procedure for building chains \(S\) and \(T\) and set \(U\)
- Finds classical cuts
- Very fast
- No guarantees
Improving the L-Shaped Method (cont’d)

LBF Strategies (cont’d)

Separation procedures (cont’d)

Exact separation approach:

- Using the graph induced by $x'$ find all connected components (chains and sets)
- Finds classical cuts
- Finds improved cuts
- Slower
- Exact separation
Improving the L-Shaped Method (cont’d)

LBF Strategies (cont’d)

Results

Test problems:

- Problem generator ⇒ Laporte, Louveaux and Vanhamme (2002)
- Instances:

  | $m = 2$ | $n = 60, 70, 80, 90$ | $\bar{f} = 90\%, 95\%, 100\%$ |
  | $m = 3$ | $n = 50, 60, 70, 80$ | $\bar{f} = 80\%, 85\%, 90\%$ |
  | $m = 4$ | $n = 20, 30, 40, 50$ | $\bar{f} = 75\%, 80\%, 85\%$ |

  - In each case, 10 instances were generated (total: 360 instances)

Algorithms:

- Heu. ⇒ L-Shaped + heuristic separation for LBF cuts
- Exa. ⇒ L-Shaped + exact separation for LBF cuts
Observations:

When using the exact separation procedure

- No clear advantage when considering solution times for those instances that were solved
- More instances are solved (about 10% more instances)
- Gives an advantage when considering gaps
Research perspectives

- General separation strategy for stochastic $m$-VRP:
  - Local branching valid inequalities
  - LBF strategies
  Integrating both approaches on the general case of the problem and extend the local branching principles to produce more efficient separations strategies for the problem

- Local branching and branch-and-price?
  Integrating both solution approaches

- Extensions to other stochastic VRP

- Questions?