An Exact Algorithm for the Multi-Vehicle Routing Problem with Stochastic Demands

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Presentation Outline

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  - Model
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  - Applying the method to the Stochastic $m$-VRP
  - Limitations of the Method

- Improving the L-Shaped Method
  - Local Branching Cuts
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    - Application to the Stochastic $m$-VRP
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Improving the L-Shaped Method (cont’d)

- LBF Strategies
  - Partial Routes
  - Classical Cuts
  - Improved Cuts
  - Separation procedures
  - Results

- Research Perspectives
**m-VRP with Stochastic Demands**

- **General Description**
  - Stochastic \( m \)-VRP: a fleet of \( m \) capacitated vehicles must deliver (collect) unknown demands to (from) a set of customers:

    \[ G(V, E): \text{an undirected graph,} \]
    \[ \text{where } V = \{v_1, \ldots, v_n\} \text{ and } E = \{(v_i, v_j) : v_i, v_j \in V, i < j\} \]
    \[ C = (c_{ij}) \text{ travel cost matrix} \]
    \[ v_1 = \text{depot} \]
    \[ D = \text{capacity of each vehicle} \]

  - Demands are revealed when vehicle arrives at a client given location:

    \[ \xi_i = \text{demand of client } i, \text{ where } i = 2, \ldots, n \]

  - Recourse \( \Rightarrow \) return to depot if vehicle cannot fulfill customer demand
m-VRP with Stochastic Demands (cont’d)

- General Description (cont’d)
  - Apply the *a priori* approach:
    Bertsimas, Jaillet and Odoni (1990)

<table>
<thead>
<tr>
<th>1(^{st}) Stage</th>
<th>General Case</th>
<th>m-VRP</th>
</tr>
</thead>
<tbody>
<tr>
<td>Build a plan</td>
<td>find <em>m</em> routes that visit all clients once</td>
<td></td>
</tr>
<tr>
<td>↓</td>
<td>↓</td>
<td></td>
</tr>
<tr>
<td>2(^{nd}) Stage</td>
<td>Use the plan</td>
<td>Run the routes</td>
</tr>
<tr>
<td></td>
<td>for a given time</td>
<td>apply recourse actions</td>
</tr>
</tbody>
</table>

- Objective:
  - Routing cost + Average recourse cost

- Constraints:
  - Restrict the total expected demand routes:
    The expected demand of each route $\leq D$
  - Obtain more balanced routes
$$m$$-VRP with Stochastic Demands (cont’d)

Model

Variables:
\[
\begin{align*}
    x_{ij} &= \{0, 1\} \quad \text{for } i, j > 1 \\
    x_{1j} &= \{0, 1, 2\} \quad \text{for } j > 1
\end{align*}
\]

Objective:
\[
\min \sum_{i<j} c_{ij} x_{ij} + Q(x)
\]

Constraints:
\[
\begin{align*}
    \sum_{j=2}^{n} x_{1j} &= 2m \\
    \sum_{i<k} x_{ik} + \sum_{j>k} x_{kj} &= 2, \quad k = 2, \ldots, n \\
    \sum_{v_i, v_j \in S} x_{ij} &\leq |S| - \left[ \sum_{v_i \in S} E(\xi_i) / D \right], \quad S \subset V \setminus \{v_1\}, 2 \leq |S| \leq n - 2 \\
    x_{ij} &= \{0, 1\}, \quad i, j > 1 \\
    x_{1j} &= \{0, 1, 2\}, \quad j > 1
\end{align*}
\]
Related Work

- Gendreau, Laporte and Séguin (1995):
  - 1st implementation of the L-Shaped algorithm on the problem

- Hjorring and Holt (1999):
  - Problem considered: 1-VRP with stochastic demands
  - New cuts ⇒ partial routes
  - Improvement of the general lower bound

- Laporte, Louveaux and Van hamme (2002):
  - Improvement of the general lower bound for the $m$-VRP with stochastic demands
  - Generalization of the partial route cuts
  - Greedy separation procedure for cuts

- Christiansen and Lysgaard (2007):
  - 1st branch-and-price algorithm for the $m$-VRP with stochastic demands
**General Solution Approach**

- 0-1 Integer L-Shaped Algorithm
  - Method proposed by Laporte and Louveaux (1993)
  - Variant of branch-and-cut

Assumption 1: $Q(x)$ is computable

Assumption 2: There exists a finite value $L = \text{general lower bound}$ for the recourse function

Let $x \in X = \overline{X} \cap \{0, 1\}^n$ define the 1st stage decisions

Define *current master problem*:

\[
\begin{align*}
\text{Min} \quad & c^T x + \Theta \\
\text{s.t.} \quad & Ax = b \\
& D_k x \geq d_k, \quad k = 1, \ldots, s, \\
& E_l x + \Theta \geq e_l, \quad l = 1, \ldots, t, \\
& 0 \leq x \leq 1, \quad \Theta \in \mathbb{R}.
\end{align*}
\]
General Solution Approach (cont’d)

0-1 Integer L-Shaped Algorithm (cont’d)

(3): Feasibility cuts ⇒ induce feasible values of \( x \)

Valid set: \( \exists s \) such that \( x \in X \) if and only if \( \{ D_k x \geq d_k, k = 1, \ldots, s \} \)

Problem dependent

(4): Optimality cuts ⇒ express possible feasible values of \( Q(x) \)

Valid set: if: \( \forall x \in X, (x, \Theta) \in \{(x, \Theta) \mid E_l x + \Theta \geq e_l, l = 1, \ldots, t\} \)
implies \( \Theta \geq Q(x) \)

Let the \( r^{th} \) feasible solution be:

- \( x_i = 1, i \in S_r \) and \( x_i = 0, i \notin S_r \)
- \( \Theta_r = \) recourse value

\[
\Theta \geq (\Theta_r - L) \left( \sum_{i \in S_r} x_i - \sum_{i \notin S_r} x_i \right) - (\Theta_r - L)(|S_r| - 1) + L
\]
General Solution Approach (cont’d)

0-1 Integer L-Shaped Algorithm (cont’d)

Algorithm:

**Step 0**: Set \( \nu = 0 \), \( \Theta \geq L \) and \( z = \infty \)

**Step 1**: Select a pendant node from the list. If none exists STOP

**Step 2**: Set \( \nu = \nu + 1 \) and solve current master problem \( \Rightarrow (x^\nu, \Theta^\nu) \)

**Step 3**: Search for violated feasibility cuts:

- If one is found \( \Rightarrow (3) + \) current master problem, return to **Step 2**
- Otherwise \( \Rightarrow \) If \( c^\top x^\nu + \Theta^\nu \geq z \) fathom node, return to **Step 1**

**Step 4**: If \( x^\nu \) is not integer \( \Rightarrow \) Branch, return to **Step 1**

**Step 5**: Compute \( Q(x^\nu) \), set \( z^\nu = c^\top x^\nu + Q(x^\nu) \)

- If \( z^\nu < z \) \( \Rightarrow \) \( z = z^\nu \)

**Step 6**:

- If \( \Theta^\nu \geq Q(x^\nu) \) \( \Rightarrow \) fathom node, return to **Step 1**
- Otherwise \( \Rightarrow (4) + \) current master problem, return to **Step 2**
General Solution Approach (cont’d)

- Applying the Method to Stochastic $m$-VRP

  **Assumptions:**

  Assumption 1: $Q(x)$ is computable

  Dror, Laporte and Trudeau (1989)

  Given a feasible solution $x$:

  \[
  Q(x) = \sum_{k=1}^{m} \min\{Q_{k,1}, Q_{k,2}\}
  \]

  Given additional assumptions:

  - Goods are divisible: $\xi_j \sim N(\mu_j, \sigma_j), j = 2, \ldots, n$
  - Demands are independent

  Assumption 2: there exists a finite value $L$

  Laporte, Louveaux and Van hamme (2002) $\Rightarrow$ measure $L$ based on the probability of failure on each route taken separately
General Solution Approach (cont’d)

- Applying the Method to Stochastic \( m \)-VRP (cont’d)
  - Feasibility cuts \( \Rightarrow \) induce feasible values of \( x \)

A priori solution \( x \) \( \Rightarrow \) a set of \( m \) routes such that the expected demand of each route \( \leq D \)

Apply cutting strategies from the deterministic version:

Considering: \( \xi_i = \mu_i \), for \( i = 2, \ldots, n \)

Lysgaard, Letchford and Eglese (2004):

- Capacity inequalities
- Framed capacity inequalities
- Strengthened comb inequalities
- Multistar and partial multistar inequalities
- Hypotour inequalities
General Solution Approach (cont’d)

- Limitations of the Method
  - Main problem: approximation of $Q(x)$
  - Information provided by optimality cuts:
    - very local
    - (4) only bound $Q(x)$ for the feasible solutions encountered
  - In *current master problem* the value of recourse:
    - $L$
    - Subset of optimality constraints
  - Risk of enumeration
Improving the L-Shaped Method

Local Branching Cuts

General Approach

Method proposed by Fischetti and Lodi (2003)
Based on the use of generic solvers (CPLEX)

Separation of the feasible region:

- let \( x^0 \) = feasible solution to the original problem,
- let \( N_1 = \{1, \ldots, n_1\} \) and \( S_0 = \{j \in N_1 \mid x^0_j = 1\} \),
- let \( \kappa \) = positive integer.
- The Hamming distance:

\[
\Delta(x, x^0) = \sum_{j \in S_0} (1 - x_j) + \sum_{j \in N_1 \setminus S_0} x_j
\]

- Left branch: \( \Delta(x, x^0) \leq \kappa \)
- Right branch: \( \Delta(x, x^0) \geq \kappa + 1 \)
Improving the L-Shaped Method (cont’d)

Local Branching Cuts (cont’d)

General Approach (cont’d)

Local Branching subproblem:

$$\Delta(x, x^0) \leq \kappa$$
Improving the L-Shaped Method (cont’d)

Local Branching Cuts (cont’d)

General Approach (cont’d)

Possible outcomes for the LB subproblem:
- subproblem is feasible (we obtain a solution \( x^1 \))
- subproblem is infeasible

If the subproblem is infeasible
- increase the size of the subproblem

If subproblem is feasible and \( c^\top x^1 + Q(x^1) \geq c^\top x^0 + Q(x^0) \)
- add \( \Delta(x, x^1) \geq 1 \)
- increase the size of the subproblem

Otherwise (i.e., \( c^\top x^1 + Q(x^1) < c^\top x^0 + Q(x^0) \))
- branching strategy
Subproblem size increase:

\[ \Delta(x, x^0) \leq \kappa + \lceil \frac{\kappa}{2} \rceil \]
Improving the L-Shaped Method (cont'd)

- Local Branching Cuts (cont'd)
  - General Approach (cont'd)

Branching step:

\[
\Delta(x, x^0) \leq \kappa \\
\Delta(x, x^0) \geq \kappa + 1
\]

\[
\Delta(x, x^1) \leq \kappa \\
\vdots
\]
Improving the L-Shaped Method (cont’d)

- Local Branching Cuts (cont’d)

- Valid Inequalities

Define the two following subproblems:

\[(P_n) \quad \text{Min} \quad c^\top x + Q(x) \quad (\overline{P}_n) \quad \text{Min} \quad c^\top x + Q(x)\]
\[\text{s.t} \quad Ax = b \quad \text{s.t} \quad Ax = b\]
\[\Delta(x, x^i) \geq \kappa_i, \quad i \in I^n \quad \Delta(x, x^i) \geq \kappa_i, \quad i \in I^n\]
\[\Delta(x, x^n) \leq \kappa_n \quad \Delta(x, x^n) \geq \kappa_n + 1\]
\[x \in X \quad x \in X,\]

- \(I^n\) = set of 0-1 vectors that may or may not represent feasible 1\(^{st}\) stage solutions

- \(P_n\) for \(n = 1, \ldots, m\) ⇒ a local branching descent

- \(\Theta_n\) = a lower bound on \(Q(x)\) for the subregion associated with \(P_n, n = 1, \ldots, m\)
Redefine the current master problem in the following way:

\[
\begin{align*}
\text{Min} & \quad c^\top x + \Theta \\
\text{s.t.} & \quad Ax = b \\
& \quad D_k x \geq d_k, \ k = 1, \ldots, s, \\
& \quad E_l x + \Theta \geq e_l, \ l = 1, \ldots, t, \\
& \quad \Delta(x, x^i) \geq 1, \ i \in \bigcup_{n=1}^{m} I^n, \\
& \quad 0 \leq x \leq 1, \ \Theta \in \mathbb{R}.
\end{align*}
\]

Constraint \(\Delta(x, x^i) \geq 1\) eliminates locally vector \(x^i\)

- If \(x^i\) is feasible \(\Rightarrow\) (10) is a weak optimality cut
- If \(x^i\) is infeasible \(\Rightarrow\) (10) serves as a feasibility cut
Improving the L-Shaped Method (cont’d)

- Local Branching Cuts (cont’d)

- Valid Inequalities (cont’d)

**Proposition**

Let $P_n, n = 1, \ldots, m$, define a local branching descent and $\Theta_n, n = 1, \ldots, m$, be valid lower bounds on the recourse value for each of the subproblems in the descent, then the following system of equations defines a set of valid inequalities for problem (6)-(11):

\begin{align*}
\Theta & \geq L + (\Theta_n - L)w_n, \ n = 1, \ldots, m \tag{12} \\
\kappa_n - \Delta(x, x^n) & \leq n_1 \sum_{j=1}^{n} w_j, \ n = 1, \ldots, m \tag{13} \\
\Delta(x, x^n) - \kappa_n & \leq (1 - w_n)n_1, \ n = 1, \ldots, m \tag{14} \\
\sum_{j=1}^{m} w_j & \leq 1 \tag{15} \\
w_n & \in \{0, 1\}, \ n = 1, \ldots, m \tag{16}
\end{align*}
Improving the L-Shaped Method (cont’d)

- Local Branching Cuts (cont’d)

- Valid Inequalities (cont’d)

System (12)-(16) bounds the value of $Q(x)$ following the local branching descent.

If $\hat{x}$ is a solution to the current master problem, then a valid lower bound on $Q(\hat{x})$ is provided in the first $P_n, n = 1, \ldots, m$ for which $\hat{x}$ is feasible.

Advantages:

- If $\Theta_n > L, n = 1, \ldots, m$, then (12)-(18) offers a better description.

- (12)-(16) bounds $Q(x)$ in different subregions of $X$.

Limitations:

- System of inequalities which makes the current master problem harder to solve.
Improving the L-Shaped Method (cont’d)

Local Branching Cuts (cont’d)

Application to the Stochastic $m$-VRP

**Case**: Stochastic 1-VRP:

Separation Strategy:

Route building:

Lower bounding functionnals:
- Partial route cuts ⇒ Hjorring and Holt (1999)

Local branching descent:
- Tightening the lower bound
Improving the L-Shaped Method (cont’d)

Local Branching Cuts (cont’d)

Application to the Stochastic $m$-VRP (cont’d)

Implementing local branching:

- Global cuts ⇒ *current master problem* (root node)
- Local cuts ⇒ *current master problem* (subproblem in search tree)
- Lower bound ⇒ *current master problem* + integrality + separation procedures

**Note:**

Global cuts can be used throughout the solution process but local cuts can only be used in the node of the search tree of the associated subproblem.
Improving the L-Shaped Method (cont’d)

Local Branching Cuts (cont’d)

Results

Test problems:

- Problem generator ⇒ Hjorring and Holt (1999)
- Problems of size $n = 20, 30, 40, \ldots, 90$ were generated
- For each size, five instances were generated for which $f = 95\%, 97.5\%, \ldots, 110\%$ (total: 280 instances)

$$\bar{f} = \frac{\sum_{i=1}^{n} \mu_i}{D} \Rightarrow \text{filling coefficient}$$

Algorithms:

- Standard: L-shaped without local branching
- LB: L-shaped + local cuts ($m = 3$ and $\kappa = 4, 6$ and 8)
- LB1: L-shaped + global cuts ($m = 3$ and $\kappa = 4, 6$ and 8)

Runs: maximum time of 1200 seconds
Improving the L-Shaped Method (cont’d)

Local Branching Cuts (cont’d)

Results (cont’d)

Standard vs. LB-4

On easy instances (< 60): the two algorithms are equivalent
On medium instances ([60, 1200]): 30 instances

<table>
<thead>
<tr>
<th>Times</th>
<th>Standard</th>
<th>LB-4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average</td>
<td>359.27 sec.</td>
<td>35.35 sec.</td>
</tr>
<tr>
<td>Total</td>
<td>10778.15 sec.</td>
<td>1060.53 sec.</td>
</tr>
</tbody>
</table>

On hard instances: 74 instances

- 24 instances solved by LB-4
- Average gap: Standard 3.42% vs. LB-4 2.04%

L-shaped algorithm is either faster or obtains better results when local branching is applied
Improving the L-Shaped Method (cont’d)

Local Branching Cuts (cont’d)

Results (cont’d)

LB vs. LB1

Variations in \( \kappa \)

Number of instances solved using LB:

- LB-4: 230
- LB-6: 231
- LB-8: 231

Number of instances solved using LB1:

- LB1-4: 217
- LB1-6: 223
- LB1-8: 221
Improving the L-Shaped Method (cont’d)

Local Branching Cuts (cont’d)

Results (cont’d)

LB vs. LB1 (cont’d)

Variations in maximum time

<table>
<thead>
<tr>
<th>Max. Time</th>
<th>LB</th>
<th>LB1</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>sol.</td>
<td>not sol.</td>
</tr>
<tr>
<td>1200</td>
<td>12/228.24</td>
<td>32/2.42%</td>
</tr>
<tr>
<td>2400</td>
<td>15/595.33</td>
<td>31/2.49%</td>
</tr>
<tr>
<td>3600</td>
<td>17/884.89</td>
<td>29/2.44%</td>
</tr>
<tr>
<td>4800</td>
<td>18/1078.25</td>
<td>29/2.52%</td>
</tr>
<tr>
<td>6000</td>
<td>18/1078.25</td>
<td>29/2.45%</td>
</tr>
</tbody>
</table>

Table: Results on hard instances: LB vs. LB1
Improving the L-Shaped Method

- LBF Strategies

Initially proposed by Hjöring and Holt (1999):
Bound the value of $Q(x)$ using Partial route cuts

- Partial Route

Illustration:

A series of chains and unstructured sets that are:
- Connected
- Begin and end at the depot
Let:

\[ S = (v_1, \ldots, v_S) \Rightarrow \text{Chain} \]

\[ T = (v_1, \ldots, v_T) \Rightarrow \text{Chain} \]

\[ U \Rightarrow \text{unstructured set such that:} \]

\[ U \cap S = \{v_S\} \]

\[ U \cap T = \{v_T\} \]

\[ R = S \cup T \cup U \]
Improving the L-Shaped Method (cont’d)

LBF Strategies (cont’d)

Classical Cuts (cont’d)

Bound the partial route:

Create node $v_0$ such that:

- $\xi_0 = \sum_{v_i \in U \setminus \{v_S, v_T\}} \xi_i$
- $c_{10} = \min_{v_i \in U \setminus \{v_S, v_T\}} \{c_{1i}\}$

Using node $v_0$, consider the following route:

$$(v_1, \ldots, v_S, v_0, v_T, \ldots, v_1)$$

By calculating $Q(x)$ this route, one obtains a lower bound $P$ on all routes that share chains $S$ and $T$ and where set $U$ is undefined.
Improving the L-Shaped Method (cont’d)

LBF Strategies (cont’d)

Classical Cuts (cont’d)

To obtain the cut:

Set: \( W(x) = \sum_{(v_i,v_j)\in S} x_{ij} + \sum_{(v_i,v_j)\in T} x_{ij} + \sum_{v_i,v_j\in U} x_{ij} - |R| + 1 \)

For 1 vehicle:

\[ \Theta \geq L + (P - L)W(x) \]

For \( m \) vehicles:

- Case where \( r \) partial routes
  
  \[ \Theta \geq L + (P - L)\left( \sum_{h=1}^{r} W_h(x) - r + 1 \right) \]
Let:

\[ S^1 = (v_1, \ldots, v_{S^1}) \Rightarrow \text{Chain no.1} \]
\[ S^2 = (v'_S, \ldots, v''_{S^2}) \Rightarrow \text{Chain no.2} \]
\[ S^3 = (v_1, \ldots, v_{S^3}) \Rightarrow \text{Chain no.3} \]
\[ U^1 \Rightarrow \text{Unstructured set no.1} \]
\[ U^2 \Rightarrow \text{Unstructured set no.2} \]
Improving the L-Shaped Method (cont’d)

LBF Strategies (cont’d)

Improved Cuts (cont’d)

Bound the partial:

Create nodes $v_{01}$ and $v_{02}$ such that:

- $\xi_{01} = \sum_{v_i \in U^1 \setminus \{v_{S1}, v'_{S2}\}} \xi_i$
- $c_{101} = \min_{v_i \in U^1 \setminus \{v_{S1}, v'_{S2}\}} \{c_{1i}\}$
- $\xi_{02} = \sum_{v_i \in U^2 \setminus \{v''_{S2}, v_{S3}\}} \xi_i$
- $c_{102} = \min_{v_i \in U^2 \setminus \{v''_{S2}, v_{S3}\}} \{c_{1i}\}$

Using nodes $v_{01}$ and $v_{02}$, consider the following route:

$(v_1, \ldots, v_{S1}, v_{01}, v'_{S2}, \ldots, v''_{S2}, v_{02}, \ldots, v_1)$

By calculating $Q(x)$ on this route, one obtains a lower bound $\hat{P}$ on all routes that share all previous chains and where all previous sets are undefined.
Improving the L-Shaped Method (cont’d)

LBF Strategies (cont’d)

Improved Cuts (cont’d)

To obtain the cut:

Set: \( \hat{\mathcal{W}}(x) = 3 \sum_{k=1}^{3} \sum_{(v_i,v_j) \in S^k} x_{ij} + 2 \sum_{k=1}^{2} \sum_{v_i,v_j \in U^k} x_{ij} - |R| + 1 \)

For 1 vehicle:
\[ \Theta \geq L + (\hat{P} - L)\hat{\mathcal{W}}(x) \]

For \( m \) vehicles:
\[ \Theta \geq L + (\hat{P} - L) \left( \sum_{h=1}^{r} \hat{\mathcal{W}}_h(x) - r + 1 \right) \]

Note: the lower bound provided by the cut is improved given that \( \hat{P} \geq P \)
Improving the L-Shaped Method (cont’d)

LBF Strategies (cont’d)

Separation procedures

A solution \((x^\nu, \Theta^\nu)\) can be eliminated using and LBF cut if:

- \(\exists h\) such that \(W_h(x^\nu) = 1\) or \(\hat{W}_h(x^\nu) = 1\)
- \(P \geq \Theta^\nu\) or \(\hat{P} \geq \Theta^\nu\)

Heuristic separation approach:

- Laporte, Louveaux and Van hamme (2002)
- Greedy procedure for building chains \(S\) and \(T\) and set \(U\)
- Finds classical cuts
- Very fast
- No guarantees
Improving the L-Shaped Method (cont’d)

- LBF Strategies (cont’d)
  - Separation procedures (cont’d)

  **Exact separation approach:**
  - Using the graph induced by $x^\nu$ find all connected components (chains and sets)
  - Finds classical cuts
  - Finds improved cuts
  - Slower
  - Exact separation
Improving the L-Shaped Method (cont’d)

LBF Strategies (cont’d)

Results

Test problems:

- Problem generator ⇒ Laporte, Louveaux and Van hamme (2002)
- Instances:

<table>
<thead>
<tr>
<th>m = 2</th>
<th>n = 60, 70, 80, 90</th>
<th>( \bar{f} = 90%, 95%, 100% )</th>
</tr>
</thead>
<tbody>
<tr>
<td>m = 3</td>
<td>n = 50, 60, 70, 80</td>
<td>( \bar{f} = 80%, 85%, 90% )</td>
</tr>
<tr>
<td>m = 4</td>
<td>n = 20, 30, 40, 50</td>
<td>( \bar{f} = 75%, 80%, 85% )</td>
</tr>
</tbody>
</table>

- In each case, 10 instances were generated (total: 360 instances)

Algorithms:

- Heu. ⇒ L-Shaped + heuristic separation for LBF cuts
- Exa. ⇒ L-Shaped + exact separation for LBF cuts
Improving the L-Shaped Method (cont’d)

LBF Strategies (cont’d)

Results (cont’d)

<table>
<thead>
<tr>
<th>m</th>
<th>( T )</th>
<th>sol. / sol.</th>
<th>sol. / not sol.</th>
<th>not sol. / sol.</th>
<th>not sol. / not sol.</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.90</td>
<td>39 445.27 40.87</td>
<td>1 9234.19 1.41%</td>
<td>0 - -</td>
<td>0 - -</td>
</tr>
<tr>
<td></td>
<td>0.95</td>
<td>29 1049.04 1300.24</td>
<td>0 - -</td>
<td>4 1.52% 1654.74</td>
<td>6 1.92% 1.67%</td>
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<td>27 2.32% 2.14%</td>
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<td>32 160.51 465.63</td>
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<td>14 4.06% 4.22%</td>
</tr>
</tbody>
</table>

Observations:

When using the exact separation procedure

- No clear advantage when considering solution times for those instances that were solved
- More instances are solved (about 10% more instances)
- Gives and advantage when considering gaps
Research perspectives

- General separation strategy for stochastic $m$-VRP:
  - Local branching valid inequalities
  - LBF strategies
  Integrating both approaches on the general case of the problem and extend the local branching principles to produce more efficient separation strategies for the problem

- Local branching and branch-and-price?
  Integrating both solution approaches

- Extensions to other stochastic VRP

- Questions?